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Short communication

Improved explicit equations for estimation of the friction factor in rough and smooth pipes

Eva Romeo, Carlos Royo, Antonio Monzón*

Department of Chemical and Environmental Engineering, Faculty of Science, University of Zaragoza, Zaragoza 50009, Spain

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Abstract

The most common correlations for calculating the friction factor in rough and smooth pipes are reviewed in this paper. From these correlations, a series of more general equations has been developed making possible a very accurate estimation of the friction factor without carrying out iterative calculus. The calculation of the parameters of the new equations has been done through non-linear multivariable regression. The better predictions are achieved with those equations obtained from two or three internal iterations of the Colebrook–White equation. Of these, the best results are obtained with the following equation:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7065} - \frac{5.0272}{Re}\log\left(\frac{\varepsilon/D}{3.827} - \frac{4.567}{Re}\log\left(\left(\frac{\varepsilon/D}{7.7918}\right)^{0.9924} + \left(\frac{5.3326}{208.815 + Re}\right)^{0.9345}\right)\right)\right).$$

Keywords: Friction factor; Colebrook–White equation; Darcy–Weisbach equation; Head loss in pipes; Non-linear regression

1. Introduction

The energy loss due to friction undergone by a Newtonian liquid flowing in a pipe is usually calculated through the Darcy–Weisbach equation [1]:

$$h_f = f\frac{L}{D}\frac{u^2}{2g} \tag{1}$$

In this equation f is the so-called Moody or Darcy friction factor (f_M or f_D, respectively) [1] which, from the above equation, is calculated as follows:

$$f_M = f_D = \frac{D}{L}\frac{gh_f}{\frac{1}{2}u^2} = \frac{D}{L}\frac{\Delta P}{\frac{1}{2}\rho u^2} \tag{2}$$

In addition to the Moody factor, the Fanning friction factor can also be used, which is defined as follows [2]:

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2} = \frac{1}{4}\frac{D}{L}\frac{\Delta P}{\frac{1}{2}\rho u^2} \tag{3}$$

From Eqs. (2) and (3) the relation between both friction factors is deduced: $f = f_M = f_D = 4f_F$.

The friction factor depends on the Reynolds number (Re), and on the relative roughness of the pipe, ε/D. For laminar flow ($Re < 2100$), the friction factor is calculated from the Hagen–Poiseuille equation:

$$f = \frac{64}{Re} = \frac{64\mu}{uD\rho} \tag{4}$$

For turbulent flow, the friction factor is estimated through the equation developed by Colebrook and White [3,4]

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.71} + \frac{2.52}{Re\sqrt{f}}\right) \tag{5}$$

The Colebrook–White equation is valid for Re ranging from 4000 to 10^8, and values of relative roughness ranging from 0 to 0.05. This equation covers the limit cases of smooth pipes, $\varepsilon = 0$, and fully developed turbulent flow [3,4]. For smooth pipes, Eq. (5) turns into the Prandtl–von Karman [3,4]:

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{D}\right) = -2\log\left(\frac{\varepsilon/D}{3.71}\right) \tag{6}$$

If the flow is fully developed, it is verified that $Re(\varepsilon/D)\sqrt{f} > 200$. In this case, the friction factor depends only on the

* Corresponding author. Tel.: +34-976-76-11-57; fax: +34-976-76-21-42.
E-mail address: amonzon@posta.unizar.es (A. Monzón).

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Nomenclature

a, b, c	constants defined in Eq. (13)
AIC	Akaike Information Criterion, Eq. (26)
D	internal pipe diameter (m)
f	friction factor
F	auxiliary variable used in Eq. (8) and defined as $1/\sqrt{f}$
$\bar{F}$	average value of the values of F
$F_{\mathrm{C–W}}$	value of F predicted with the Colebrook–White equation
F_{calc}	value of F calculated with the proposed model
g	gravity of acceleration (9.81 m^2/s)
h_{f}	head loss (m)
L	pipe length (m)
MSC	model selection criterion, Eq. (25)
n	number of points, Eq. (25)
NP	number of parameters, Eq. (25)
ΔP	pressure drop (Pa)
Re	Reynolds number
$Re\sqrt{f}$	Karman number $(= (D\rho/\mu)\sqrt{2D\Delta P/\rho L})$
u	mean velocity (m/s)
Y	auxiliary variable defined by Eq. (9)

Greek symbols

ε	pipe roughness (m)
μ	viscosity (kg/ms)
ρ	density (kg/m^3)
τ_{w}	shear stress (Pa)

relative roughness and can be calculated through the equation deduced by von Karman [3,4]

$$\frac{1}{\sqrt{f}} = 2\log(Re\sqrt{f}) - 0.8 = 2\log\left(\frac{Re\sqrt{f}}{2.52}\right) \tag{7}$$

Unless the Karman number, $Re\sqrt{f}$, is previously known, i.e. the pressure drop of the fluid in the pipe is known, Eqs. (5) and (7) are implicit with respect to the value of f, and are solved using numerical methods. Thus, if the auxiliary variable F is defined as $1/\sqrt{f}$, the Colebrook–White equation (Eq. (5)) can be re-written to be solved by a method of successive substitution:

$$F_{n+1} = -2\log\left(\frac{\varepsilon/D}{3.71} + \frac{2.52}{Re}F_n\right) \tag{8}$$

Alternatively, the Newton–Raphson method can be used. According to this method, again using the variable F, the calculation of the f value in Eq. (5) involves finding the root of the function Y, which is defined as

$$Y = F + 2\log\left(\frac{\varepsilon/D}{3.71} + \frac{2.52}{Re}F\right) \tag{9}$$

From the above equation, the iterations to calculate the root of the function Y are carried out through the following expression:

$$F_{n+1} = F_n - \left(\frac{Y_n}{Y'_n}\right) \tag{10}$$

where Y'_n can be evaluated as:

$$Y'_n = \left(\frac{\mathrm{d}Y}{\mathrm{d}F}\right)_n = 1 + \frac{18.7}{Re(\varepsilon/D) + 9.35F_n} \tag{11}$$

Eqs. (8) and (10) converge very rapidly, especially if there is a good initial estimation of the friction factor. For this the graph produced by Moody [1] or any of the explicit equations available in the literature can be used.

An alternative solution to the iterative methods is the direct use of an explicit equation which is precise enough to calculate the value of f directly. In the case of smooth pipes, in which f depends only on Re, Gulyani [5] provides a revision and discussion of the correlations more commonly used to estimate the friction factor. In the general case of rough tubes, numerous equations have been proposed since the 1940s. In this work, a revision of those more frequently used is presented. From these expressions, new equations are also proposed resulting in an improvement in the direct calculation of the friction factor.

1.1. Review of previous equations for calculation of the friction factor

The most widely used equations postulated since the end of the 1940s are stated below in the order of publication.

(i) In 1947, Moody [6] proposed the following empirical equation:

$$f = 0.0055\left(1 + \left(2\times10^4\frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{1/3}\right) \tag{12}$$

According to the author, this equation is valid for Re ranging from 4000 to 10^8 and values of ε/D ranging from 0 to 0.01.

(ii) Later, Wood [7] proposed the following correlation:

$$f = a + b\,Re^{c} \tag{13}$$

where $a = 0.53(\varepsilon/D) + 0.094(\varepsilon/D)^{0.225}$, $b = 88(\varepsilon/D)^{0.44}$, $c = 1.62(\varepsilon/D)^{0.134}$.

This equation is recommended for Re between 4000 and 10^7 and values of ε/D ranging from 0.00001 to 0.04.

(iii) Churchill [8], using the transport model developed by Churchill and Usagi [9], deduced the following expression:

$$e^{-1/0.869\sqrt{f}} = \frac{\varepsilon/D}{3.70} + \left(\frac{7}{Re}\right)^{0.9} \Leftrightarrow \frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.70} + \left(\frac{7}{Re}\right)^{0.9}\right) \tag{14}$$

(iv) From the von Karman–Prandtl equation (Eq. (6)), Jain [10] proposed a similar expression to that by Churchill [8]:

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{D} + \frac{21.25}{Re^{0.9}}\right) = -2\log\left(\frac{\varepsilon/D}{3.715} + \left(\frac{6.943}{Re}\right)^{0.9}\right) \quad (15)$$

This equation is recommended for Re ranging from 5000 to 10^7 and values of ε/D between 0.00004 and 0.05.

(v) Churchill [11], again using the Churchill and Usagi transport model [9], proposed the following equation valid for the whole range of Re (laminar, transition and turbulent):

$$f = 8\left(\left(\frac{8}{Re}\right)^{12} + (A+B)^{-3/2}\right)^{1/12} \quad (16)$$

where $A = [-2\log(((\varepsilon/D)/3.70) + (7/Re)^{0.9})]^{16}$, $B = (37530/Re)^{16}$.

The above expression includes the Haguen–Poiseuille equation for laminar flow ($Re < 2100$) (Eq. (4)), Eq. (14) for turbulent flow ($Re > 4000$) (term A in Eq. (16)) and the following correlation for the transition regime ($2100 < Re < 4000$) (term B in Eq. (16)) [11]

$$f = 7 \times 10^{-10}\, Re^2 \quad (17)$$

(vi) Following manipulation of Eq. (5) to obtain an implicit expression for $1/\sqrt{f}$, and substitution of this expression in the Colebrook–White equation (Eq. (5)), Chen [12] proposed the following equation:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7065} - \frac{5.0452}{Re} \times\log\left(\frac{(\varepsilon/D)^{1.1098}}{2.8257} + \frac{5.8506}{Re^{0.8981}}\right)\right) \quad (18)$$

This method involves carrying out two iterations of the Colebrook–White equation. The accuracy of the results obtained from this equation is high due to the fact that the initial estimate is good. The equation proposed by Chen is valid for Re ranging from 4000 to 4.10^8 and values of ε/D between 0.0000005 and 0.05.

(vii) Round [13] proposed the following change to the Altshul equation [14] which improves the predictions of this equation for high values of ε/D:

$$\frac{1}{\sqrt{f}} = -1.8\log\left(0.27\frac{\varepsilon}{D} + \frac{6.5}{Re}\right) \quad (19)$$

(viii) Barr [15], by a method analogous to that used by Chen [12], proposed the following expression:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.70} + \frac{4.518\log(\frac{1}{7}Re)}{Re(1+\frac{1}{29}Re^{0.52}(\varepsilon/D)^{0.7})}\right) \quad (20)$$

(ix) Zigrang and Sylvester [16] also followed the same method as that used by Chen [12], but carried out three internal iterations. They proposed the following equation:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re} \times\log\left(\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re}\log\left(\frac{\varepsilon/D}{3.7} + \frac{13}{Re}\right)\right)\right) \quad (21)$$

(x) Haaland [17] proposed a variation in the effect of the relative roughness by the following expression:

$$\frac{1}{\sqrt{f}} = -1.8\log\left(\left(\frac{\varepsilon/D}{3.70}\right)^{1.11} + \frac{6.9}{Re}\right) \quad (22)$$

(xi) Manadilli [18], using what he calls *signomial*-like equations, proposed the following expression valid for Re ranging from 5235 to 10^8, and for any value of ε/D:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.70} + \frac{95}{Re^{0.983}} - \frac{96.82}{Re}\right) \quad (23)$$

In addition, for values of Re ranged between 2100 and 5235, Manadilli [18] proposed the following expression to calculate f:

$$f = 2.82 \times 10^{-7}\, Re^{1.5} \quad (24)$$

This equation is similar to that proposed by Churchill [11] for the transition regime (Eq. (17)).

1.2. Comparison of the equations

The statistical comparison of the different equations, both those in the literature and those developed in the present work, has been carried out using the statistical parameter designated as the model selection criterion (MSC) [19] calculated by the following expression:

$$\mathrm{MSC} = \ln\left(\frac{\sum_{i=1}^{n}(F_{\mathrm{C\text{–}W}_i} - \bar{F}_{\mathrm{C\text{–}W}})}{\sum_{i=1}^{n}(F_{\mathrm{C\text{–}W}_i} - F_{\mathrm{calc}_i})}\right) - \frac{2\mathrm{NP}}{n} \quad (25)$$

As can be seen in the above equation, we estimate the value of $F\ (= 1/\sqrt{f})$ instead of the value of the friction factor. This criterion is derived from the Information Criterion of Akaike [20] and allows a direct comparison among models with a different number of parameters (NP). The Akaike Information Criterion (AIC) is defined by the following expression [20]:

$$\mathrm{AIC} = n\ln\left(\sum_{i=1}^{n}(F_{\mathrm{C\text{–}W}_i} - F_{\mathrm{calc}_i})^2\right) + 2\mathrm{NP} \quad (26)$$

The AIC attempts to represent the “information content” of a given set of parameter estimates by relating the coefficient of determination to the NP (or equivalently, the number of degrees of freedom) that were required to obtain the fit. When

Table 1
Values of MSC obtained with previous models

Authors	Equation	MSC
Moody [6]	(12)	4.639
Wood [7]	(13)	−4.019
Churchill [8]	(14)	8.980
Jain [10]	(15)	9.118
Chen [12]	(18)	12.180
Round [13]	(19)	3.067
Barr [15]	(20)	12.247
Zigrang and Sylvester [16]	(21)	12.537
Haaland [17]	(22)	8.845
Manadilli [18]	(23)	9.722

comparing two models with different numbers of parameters, this criterion places a burden on the model with more parameters to not only have a better coefficient of determination, but quantifies how much better it must be for the model to be deemed more appropriate. The AIC as defined above is dependent on the magnitude of the data points as well as the number of observations. According to this criterion, the most appropriate model is the one with the smallest value of the AIC.

The MSC will give the same rankings between models as the AIC and has been normalized so that it is independent of the scaling of the data points. For this criterion, the most appropriate model will be that with the largest MSC, because we want to maximize information content of the model. In Table 1, the MSC values for the revised equations are shown. It can be observed that the best fits correspond to those proposed by Chen [12], Barr [15] and Zigrang and Sylvester [16].

1.3. Proposed models

New equations are suggested which generalize the best of the previously proposed correlations (see Table 1) and allow estimates of the friction factor almost without error. The calculation of the parameters has been carried out by multivariable non-linear regression of the data F vs. Re and ε/D generated from the Colebrook–White equation (data referred to as $F_{\mathrm{C–W}}$). There are 20 curves of F vs. Re data (Re ranging from 3000 to 1.5×10^8) for 21 values of ε/D ranged from 0 to 0.05. With the aim of ensuring a high statistical reliability on the obtained parameters, about 500 points have been calculated in each curve ($n = 10548$ points).

1.4. Model 1

This model, Eq. (27), uses a rational function to express the influence of the Re number, and represents a generalization of the Churchill (Eq. (14)), Swamee and Jain (Eq. (15)), Round (Eq. (19)) and Haaland (Eq. (22)) equations:

$$\frac{1}{\sqrt{f}} = \left(-a_0 \log\left(\left(\frac{\varepsilon/D}{a_1}\right)^{n_1} + \left(\frac{a_2}{a_3 + Re}\right)^{n_2}\right)\right)^m \tag{27}$$

Table 2 shows the values and number of fitted parameters (NP), the MSC obtained with each of the 10 cases studied. Each case has been obtained fixing some of the following parameters of Eq. (27): n_1, n_2, a_3 and m, and carrying out the fitting, leaving free the rest of the parameters to be estimated. The results in Table 2 indicate that Eq. (27) considerably improves the results of the fittings with respect to the previous models, especially when the parameter a_3 is included (Models 1F–1J). From a statistical point of view, the best result is obtained with Model 1J (MSC $= 10.90$), although Model 1I provides almost the same fitting (MSC $= 10.75$) and is more simple to use.

1.5. Model 2

Model 2, Eq. (28), involves an extension of the Chen model (Eq. (18)), and is developed carrying out two internal iterations of the Colebrook–White equation. Taking the result of Model 1I as an initial estimation:

$$\frac{1}{\sqrt{f}} = \left(-a_0 \log\left(\frac{\varepsilon/D}{a_1} - \frac{a_2}{Re} \times \log\left(\left(\frac{\varepsilon/D}{a_3}\right)^{n_1} + \left(\frac{a_4}{a_5 + Re}\right)^{n_2}\right)\right)\right)^m \tag{28}$$

Table 2
Parameters of the sub-models obtained from Model 1

Model 1	a_0	a_1	a_2	a_3	n_1	n_2	m	NP	MSC
Model 1A	1.9092	5.2104	11.8301	0[a]	1[a]	1[a]	1[a]	3	6.332
Model 1B	2.0032	3.7253	6.9895	0[a]	1[a]	0.8992	1[a]	4	9.272
Model 1C	1.7981	3.7507	7.0470	0[a]	1.1094	1[a]	1[a]	4	9.304
Model 1D	1.8970	3.7238	6.9783	0[a]	1.0543	0.9480	1[a]	5	9.528
Model 1E	1.7096	4.7879	4.9133	0[a]	1.0610	0.9476	1.0347	6	10.010
Model 1F	1.9485	4.5071	16.0955	2497.60	1[a]	1[a]	1[a]	4	7.134
Model 1G	1.8250	3.7654	8.3846	677.4126	1.0929	1[a]	1[a]	5	10.427
Model 1H	2.0014	3.7490	8.2096	611.4563	1[a]	0.9122	1[a]	5	10.009
Model 1I	1.8900	3.7466	8.2268	626.8860	1.0572	0.9647	1[a]	6	10.750
Model 1J	1.8166	4.1165	7.0367	527.6200	1.0593	0.9619	1.0131	7	10.900

[a] These values have been fixed for each fitting.

Table 3
Parameters of the sub-models obtained from Model 2

	Model 2A	Model 2B	Model 2C	Model 2D	Model 2E	Model 2F
a_0	1.9977	1.9986	1.9993	1.9997	2.0152	2.0064
a_1	3.7360	3.7241	3.7162	3.7102	3.6415	3.6793
a_2	4.4570	4.5827	4.8294	4.8374	4.9177	4.8738
a_3	9.4923	7.8435	4.2397	4.2134	4.3578	4.2678
a_4	6.4219	8.3049	6.3098	7.7436	6.1864	7.4964
a_5	0[a]	622.67	0[a]	480.13	0[a]	420.15
n_1	1[a]	1[a]	1.0271	1.0287	1.0278	1.0287
n_2	1[a]	1[a]	0.9264	0.94457	0.92698	0.94258
m	1[a]	1[a]	1[a]	1[a]	0.99732	0.99887
NP	5	6	7	8	8	9
MSC	13.4576	14.2610	14.3708	16.0261	14.8488	16.3568

[a] These values have been fixed for each fitting.

In Table 3, the results obtained with the six cases studied are shown, Model 2C being analogous to that presented by Chen [12]. As a consequence of the structure of this function, even for the most simple case (Model 2A), the MSC values indicate that all the fittings obtained are better than those obtained with any of the cases studied with Model 1, and even better than those obtained with the Chen model. The inclusion of parameter a_5 results in a considerable improvement in the fittings (Models 2B, 2D and 2F). From the statistical point of view, the best fitting is provided by Model 2F (MSC $=$ 16.36), although Model 2D provides almost the same fitting (MSC $=$ 16.03) and is more simple to use.

1.6. Model 3

Model 3, Eq. (29), is an extension of the Zigrang and Sylvester model, Eq. (21), and is obtained carrying out three internal iterations in the Colebrook–White equation, taking Model 1I as an initial estimate (see Table 2):

$$\frac{1}{\sqrt{f}} = \left(-a_0 \log\left(\frac{\varepsilon/D}{a_1} - \frac{a_2}{Re}\log\left(\frac{\varepsilon/D}{a_3} - \frac{a_4}{Re} \times \log\left(\left(\frac{\varepsilon/D}{a_5}\right)^{n_1} + \left(\frac{a_6}{a_7 + Re}\right)^{n_2}\right)\right)\right)\right)^m \tag{29}$$

The results obtained for the five cases studied with this model are shown in Table 4. From these results, the improvement of the fitting provided by Model 3A with respect to the Zigrang and Sylvester model is obvious, even though they have the same NP. This difference is due to the estimation of parameters a_2, a_4 and a_6 carried out in the work of Zigrang and Sylvester [16]. Again, the structure of Eq. (29) leads to all the obtained fittings being better than those obtained with Models 1 and 2. As in the previous cases, the insertion of parameter a_7 (Models 3D and 3E), provides a considerable improvement in the fittings. From a statistical point of view, Models 3D and 3E provide a very similar

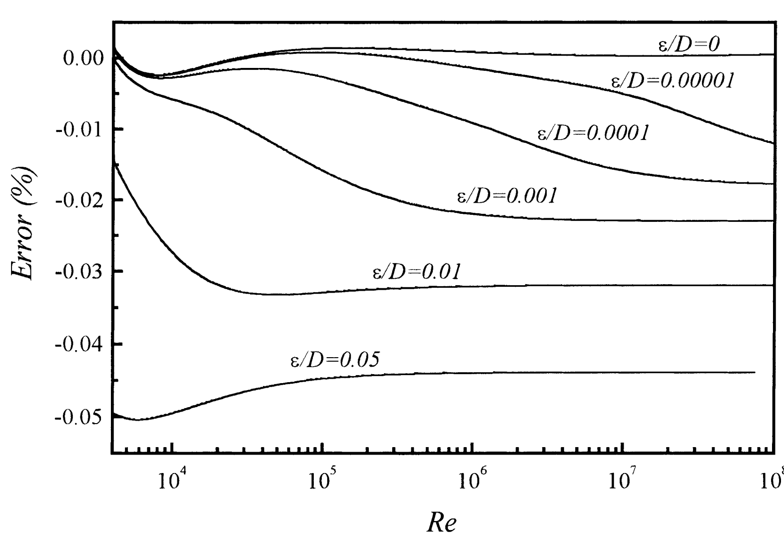

Fig. 1. Percentage of error in the estimation of the friction factor with Model 3E. Influence of Re and ε/D.

Table 4
Parameters of the sub-models obtained from Model 3

	Model 3A	Model 3B	Model 3C	Model 3D	Model 3E
a_0	1.9999	1.9999	2.0002	2.0000	2.0000
a_1	3.7073	3.7072	3.7062	3.7069	3.7065
a_2	5.0120	5.0167	5.0163	5.0293	5.0272
a_3	3.8855	3.8602	3.8713	3.7924	3.8270
a_4	4.0469	4.2537	4.2139	4.5203	4.5670
a_5	14.4349	9.9399	10.5465	6.8435	7.7918
a_6	3.9851	4.2094	4.1080	5.5917	5.3326
a_7	0[a]	0[a]	0[a]	210.362	208.815
n_1	1[a]	1.0046	1.0038	0.9936	0.9924
n_2	1[a]	0.9642	0.9678	0.9371	0.9345
m	1[a]	1[a]	1.0000	1[a]	1.0000
NP	7	9	10	10	11
MSC	20.536	20.898	20.935	22.108	22.111

[a] These values have been fixed for each fitting.

fitting, the values of the parameters being almost equal. In Fig. 1, are presented the percentages of error obtained with Model 3E, in the estimation of the friction factor values. In this figure, it can be seen that in all the cases the errors are very small (always less than 0.05% of error) and how, for a given value of *Re*, an increase of relative roughness (ε/D) increases the error of the estimation.

2. Conclusions

From the correlations shown in the literature, a series of more general equations has been developed making possible a very accurate estimation of the friction factor without carrying out iterative calculus. The best predictions are achieved with those equations obtained from two or three internal iterations of the Colebrook–White equation. This method could be extended to a larger number of internal iterations but the expressions obtained would be too complex. From a statistical point of view, the following equation corresponding to Model 3E provides the best results:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7065} - \frac{5.0272}{Re}\log\left(\frac{\varepsilon/D}{3.827} - \frac{4.567}{Re} \times\log\left(\left(\frac{\varepsilon/D}{7.7918}\right)^{0.9924} + \left(\frac{5.3326}{208.815+Re}\right)^{0.9345}\right)\right)\right)$$

The ranges of application of this equation are *Re* between 3000 and 1.5×10^8 and ε/D between 0 and 0.05.

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